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Symmetry breaking induced by a magnetic field in a quasi-two-dimensional d-wave superconductor

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Abstract. A model of quasi-two-dimensional *d*-wave superconductor, with strong nesting properties of the Fermi surface is considered. The orbital effect of a moderate magnetic field applied perpendicularly to the conducting planes is studied in the mean field approximation. It is shown that the field can induce a time reversal symmetry breaking SDW order coexisting with the superconducting order and can open a gap over the whole Fermi surface. The anomalies recently observed in the heat conductivity in $Bi_2Sr_2CaCu_2O_{8+\delta}$ might be ascribed to this effect.

PACS. 74.20.Mn Nonconventional mechanisms (spin fluctuations, polarons and bipolarons, resonating valence bond model, anyon mechanism, marginal Fermi liquid, Luttinger liquid, etc.) – 75.30.Fv Spin-density waves – 74.72.Hs Bi-based cuprates

1 Introduction

During the last few years, a great deal of efforts has been devoted to the determination of the high T_c superconductors order parameter symmetry. The pairing symmetry provides clues to the understanding of the superconducting pairing mechanism, which is essential for the development of the theory of high temperature superconductivity in cuprates. Several authors have given experimental evidences for a *d*-wave order parameter in various high T_c superconductors, in particular in the cuprate $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi2212) [1–4].

The presence of nodes in the superconducting gap allows the existence of quasiparticles, even at low temperature. This peculiarity offers the interesting opportunity of testing the quasiparticle properties. Recent experimental results have brought a large number of new insights concerning quasiparticle behaviour, in particular in a magnetic field [5,6]. It is well-known that a two-dimensional system of quasiparticles exhibits original properties when a magnetic field is applied perpendicularly to its plane. The orbital effect of the field makes the quasiparticle motion strictly one-dimensional. The result is a Spin Density Wave (SDW) zero-sound type instability induced by the field and the opening of a gap at the Fermi level [7,8]. In quasi-two-dimensional conductors, the same effect still exists above a threshold field related to the transverse coupling. The intensity of the relevant magnetic field is determined by the Fermi surface geometry and such a field can be available in the laboratory only when this geometry exhibits good nesting. Such is the case of various high T_c superconductors, such as Bi2212. It is therefore interesting to study the effect of the field in a quasi-two-dimensional conductor, in the case where the ground state exhibits *d*-wave superconductivity, with large values of the critical temperature and critical fields. Such study asks the questions of the competition between the *d*-wave superconductivity existing in the absence of the field and the tendency to SDW ordering induced by the field, the possible coexistence between these two different types of ordering, the symmetry breaking in the presence of the field and the behaviour of the gap at the Fermi energy in the reciprocal space.

Krishana *et al.* [5] have, indeed, reported a series of high-resolution measurements of thermal conductivity κ in the cuprate Bi2212 that reveal a surprising feature of the quasi-particles heat current. These measurements exhibit a phase transition induced by a magnetic field and characterized by a kink in the thermal conductivity as a function of field strength, followed by a flat plateau. In the high field state, the quasiparticle current is still zero which means that the superconducting order parameter still exists. However, if the interpretation of the data by a disappearance of the quasiparticles is correct [5,9], the new state is necessarily characterized by a new order parameter and a new symmetry breaking [10]. Indeed, the Fermi surface of Bi2212 displays extremely good nesting properties, probably the best among all the known

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high T_c superconductors [11,12]. Therefore, if the superconducting state would not exist, we would expect that above a moderate threshold field, a field induced SDW occurs and breaks the time-reversal symmetry. However, we know that, in the absence of the field, the stable state at low temperature is superconducting. Therefore, we discuss, in the present work, the nature of the new high-field state and how the anomalies observed in the thermal conductivity of Bi2212 could be explained in this framework by the disappearance of the quasiparticles in the applied field.

In the following, we present our simple model of a quasi-two-dimensional d-wave superconductor. Then we study, in the mean field approximation, the phase diagram when a moderate magnetic field is applied perpendicularly to the conducting plane. We discuss, in particular, the possible coexistence of d-wave superconductivity and SDW ordering. Such a coexistence should correspond to new symmetry breaking and should open a gap on the whole Fermi surface. Finally, we comment about the possibility to ascribe to this effect the anomalies observed in the thermal conductivity of Bi2212.

2 Model

We study a nearly square two-dimensional Fermi surface with strong nesting properties, as a model for high T_c superconductors. We assume that well defined quasiparticles exist at low temperature and can be treated in a Landau-Fermi liquid approach.

We propose to treat the electron-electron interaction in a mean field approximation, with two different symmetry-breaking order parameters: the first is an effective attractive electron-electron interaction term, which breaks the gauge symmetry. We do not specify the microscopic origin of this term leading to superconductivity. The second, the origin of which is the exchange interaction, is an electron-hole coupling term, leading to a time-reversal symmetry breaking SDW state. The mean field Hamiltonian H includes the non interacting electron term H_0, the superconducting term H_s and the magnetic term H_m

$$H = H_0 + H_\mathrm{s} + H_\mathrm{m}$$

where

$$H_0 = \sum_{k,\sigma} \xi_k c^+_{k,\sigma} c_{k,\sigma}$$
$$H_\mathrm{s} = -\sum_{k,k'} g_\mathrm{s}(k,k') c^+_{k,\uparrow} c^+_{-k,\downarrow} c_{-k',\downarrow} c_{k',\uparrow}$$
$$H_\mathrm{m} = -\sum_{k,k'} g_\mathrm{m}(k,k') c^+_{k+q,\downarrow} c_{k,\uparrow} c^+_{k'-q,\uparrow} c_{k',\downarrow}$$

where ξ_k is the non interacting electron dispersion relation, $c_{k,\sigma}$ is an electron annihilation operator. The coefficients g_s and g_m are respectively the superconductivity and the magnetic coupling constants.

In the nearly square Fermi surface, we approximate the electron dispersion relation by a linearized expression for two opposite flat sheets of the Fermi surface

$$\xi(k_x, k_y) = \xi^0(k_x) + t_\perp(k_y) - \mu \tag{1}$$

where

$$\xi^0(k_x) = v_\mathrm{F}(|k_x| - k_\mathrm{F})$$
$$t_\perp(k_y) = -2t\cos(k_y b) - 2t'\cos(2k_y b)$$

where k_x and k_y are respectively the electron wave vector components perpendicular and parallel to the Fermi surface sheet and μ is the chemical potential. Here, the dispersion relation is considered as linear along k_x and $t_\perp(k_y b)$, where b is the lattice parameter along the y-direction, represents the periodic dependence on k_y. The second harmonic term t' introduces the deviation from perfect nesting of the Fermi surface. It is essential to preserve the metallic or superconducting phase stability in the absence of the applied magnetic field. This dispersion relation is strictly two-dimensional. The straightforward extension to the quasi two-dimensional case introduces a small threshold field in the phenomena studied here and will be discussed later. A similar dispersion relation applies for the other pair of parallel sheets of the Fermi surface. In this work, the Fermi velocity will be considered as a constant. The extension to a k-dependent Fermi velocity will be considered in a forthcoming paper.

The calculation will be done in the mixed representation $\left(\frac{\partial}{\partial x}, k_y\right)$. The magnetic field $\mathbf{H} = \mathbf{rot}\mathbf{A}$ will be written in the following gauge: $\mathbf{A} = (0, Hx)$. The effect of the field will be taken into account by the Peierls substitution:

$$\mathbf{k} \rightarrow \mathbf{k} - \frac{e\mathbf{A}}{c}\cdot$$

We have not taken into account the Zeeman effect energy terms in the Hamiltonian. These terms, indeed, split the Fermi surface into two pieces for up and down spins respectively. However, they cancel out when one calculates the SDW response function and order parameter, since these quantities only involve the product of Green's functions for opposite spins. Such would not be the case for a charge density wave instability, where the two Fermi surfaces would have two different nesting wave vectors, each of them involving only one half of the density of states, with a dramatic effect on the critical temperature. This is why a spin density wave rather than a charge density wave is induced by the magnetic field.

In the second part of this section, we shall first write down Gork'ov's equations for Green functions in the superconducting and SDW states in the presence of an applied magnetic field. We shall then study the mean field self-consistent conditions for superconductivity and SDW order parameters. Then, in the third part of this section, we shall calculate the thermodynamic potential corresponding to the possible states and compare their stability as a function of temperature and magnetic field. Finally, we shall propose a possible phase diagram for a set of likely physical parameters.

2.1 Gap equations

We define the corresponding time Fourier transforms of the Green functions in the presence of an applied magnetic field

$$F_{\mathrm{m}}^{+}(k,\tau-\tau') = -T_\tau\langle\,[\,c_{H,k+q\downarrow}(\tau)c_{H,k\uparrow}^{+}(\tau')\,]\,\rangle \quad (2)$$
$$F_{\mathrm{s}}^{+}(k,\tau-\tau') = -T_\tau\langle\,[\,c_{H,-k\,\downarrow}^{+}(\tau)c_{H,k\uparrow}^{+}(\tau')\,]\,\rangle \quad (3)$$

and the single particle Green's function is given by

$$G(k,\tau) = -T_\tau\langle[c_{H,k\uparrow}(\tau)c_{H,k\uparrow}^{+}(\tau')]\rangle. \quad (4)$$

The time and space Fourier transforms of these Green's functions satisfy the following coupled equations of motion

$$\begin{aligned}
&[\mathrm{i}\hbar\omega_n + \mathrm{i}v_{\mathrm{F}}\frac{\partial}{\partial x} + v_{\mathrm{F}}k_{\mathrm{F}} + \mu + t_\perp(k_y b - eHbx)]\\
&\quad\times G(x,x',k_y,\omega_n) + \Delta_{\mathrm{s}}F_{\mathrm{s}}^{+}(x,x',k_y,\omega_n)\\
&\quad + \Delta_{\mathrm{m}}F_{\mathrm{m}}^{+}(x,x',k_y,\omega_n) = \hbar\delta(x-x') \quad (5)\\
&[-\mathrm{i}\hbar\omega_n - \mathrm{i}v_{\mathrm{F}}\frac{\partial}{\partial x} + v_{\mathrm{F}}k_{\mathrm{F}} + \mu - t_\perp(-k_y b - eHbx)]\\
&\quad\times F_{\mathrm{s}}^{+}(x,x',k_y,\omega_n) - \Delta_{\mathrm{s}}^{*}G(x,x',k_y,\omega_n) = 0 \quad (6)\\
&[\mathrm{i}\hbar\omega_n - \mathrm{i}v_{\mathrm{F}}\frac{\partial}{\partial x} - v_{\mathrm{F}}k_{\mathrm{F}} - v_{\mathrm{F}}q_x + \mu\\
&\quad - t_\perp(k_y b - eHbx + Q_y b)]F_{\mathrm{m}}^{+}(x,x',k_y,\omega_n)\\
&\quad + \Delta_{\mathrm{m}}^{*}G(x,x',k_y,\omega_n) = 0 \quad (7)
\end{aligned}$$

where $p = k_y b - eHbx$, ω_n indicate Matsubara frequencies and $\mathbf{Q} = (2k_{\mathrm{F}} + q_x, Q_y)$ is the SDW vector.

It is possible to write these equations in a simpler form by expliciting phase factors in the various Green's functions and defining

$$\begin{aligned}
g &= G(x,x',k_y,\omega_n)\mathrm{e}^{-\mathrm{i}\varphi_g(x,x',k_y)}\\
f_{\mathrm{s}} &= F_{\mathrm{s}}^{+}(x,x',k_y,\omega_n)\mathrm{e}^{-\mathrm{i}\varphi_{\mathrm{s}}(x,x',k_y)}\\
f_{\mathrm{m}} &= F_{\mathrm{m}}^{+}(x,x',k_y,\omega_n)\mathrm{e}^{-\mathrm{i}\varphi_{\mathrm{m}}(x,x',k_y)}.
\end{aligned}$$

It is straightforward to show that for the following choice of the phase factors

$$\begin{aligned}
\varphi_g &= k_{\mathrm{F}}(x-x') - \frac{1}{v_{\mathrm{F}}}\int_{x'}^{x} t_\perp\left(p - \frac{u}{x_0}\right)\mathrm{d}u\\
\varphi_{\mathrm{s}} &= k_{\mathrm{F}}(x-x') - \frac{1}{v_{\mathrm{F}}}\int_{x'}^{x} t_\perp\left(p - \frac{u}{x_0}\right)\mathrm{d}u\\
&\quad - \frac{1}{v_{\mathrm{F}}}\int_{x'}^{x} t_\perp\left(-p - \frac{u}{x_0}\right)\mathrm{d}u\\
\varphi_{\mathrm{m}} &= k_{\mathrm{F}}(x-x') + \frac{1}{v_{\mathrm{F}}}\int_{x'}^{x} t_\perp\left(p - \frac{u}{x_0}\right)\mathrm{d}u\\
&\quad + \frac{1}{v_{\mathrm{F}}}\int_{x'}^{x} t_\perp\left(p - \frac{u}{x_0} + Q_y b\right)\mathrm{d}u
\end{aligned}$$

where $x_0 = \frac{c}{eHb}$ is the magnetic length, the motion equations read :

$$(\mathrm{i}\omega_n + \mathrm{i}v_{\mathrm{F}}\frac{\partial}{\partial x})g + \widetilde{\Delta}_{\mathrm{s}}f_{\mathrm{s}} + \widetilde{\Delta}_{\mathrm{m}}f_{\mathrm{m}} = \delta(x-x')\mathrm{e}^{-\mathrm{i}\varphi_g} \quad (8)$$
$$(-\mathrm{i}\omega_n - \mathrm{i}v_{\mathrm{F}}\frac{\partial}{\partial x})f_{\mathrm{s}} - \widetilde{\Delta}_{\mathrm{s}}^{*}g = 0 \quad (9)$$
$$(\mathrm{i}\omega_n - \mathrm{i}v_{\mathrm{F}}\frac{\partial}{\partial x} - v_{\mathrm{F}}q_x)f_{\mathrm{m}} + \widetilde{\Delta}_{\mathrm{m}}^{*}g = 0 \quad (10)$$

where $\widetilde{\Delta}_{\mathrm{s}}$ and $\widetilde{\Delta}_{\mathrm{m}}$ are new effective self consistent potentials defined as

$$\begin{aligned}
\widetilde{\Delta}_{\mathrm{s}}(x,k_y) &= \Delta_{\mathrm{s}}(x)\exp\left\{\frac{\mathrm{i}}{v_{\mathrm{F}}}(-2v_{\mathrm{F}}k_{\mathrm{F}} - 2\mu)(x-x')\right.\\
&\quad -\mathrm{i}\frac{x_0}{v_{\mathrm{F}}}\int_{x'}^{x}\left[t_\perp\left(k_y b - \frac{u}{x_0}\right)\right.\\
&\quad \left.\left. + t_\perp\left(-k_y b - \frac{u}{x_0}\right)\right]\mathrm{d}u\right\} \quad (11)\\
\widetilde{\Delta}_{\mathrm{m}}(x,k_y) &= \Delta_{\mathrm{m}}(x)\exp\left\{\frac{\mathrm{i}}{v_{\mathrm{F}}}(-2\mu)(x-x')\right.\\
&\quad +\mathrm{i}\frac{x_0}{v_{\mathrm{F}}}\int_{x'}^{x}\left[t_\perp\left(k_y b - \frac{u}{x_0}\right)\right.\\
&\quad \left.\left. + t_\perp\left(k_y b - \frac{u}{x_0} + Q_y b\right)\right]\mathrm{d}u\right\}. \quad (12)
\end{aligned}$$

The phase factor of $\widetilde{\Delta}_{\mathrm{m}}(x)$ is a periodic function which can be expanded on the basis of plane waves

$$\widetilde{\Delta}_{\mathrm{m}}(x) = \Delta_{\mathrm{m}}\sum_n a_n I_n(z,z')\exp\left(-\mathrm{i}n\frac{x}{x_0}\right) \quad (13)$$

where n is an integer and

$$\begin{aligned}
z &= \frac{4tx_0}{v_{\mathrm{F}}}\cos\left(\frac{Q_y b}{2}\right)\\
z' &= \frac{2t'x_0}{v_{\mathrm{F}}}\cos(Q_y b)\\
a_n(p) &= \exp(-\mathrm{i}z\sin p - \mathrm{i}z'\sin 2p + \mathrm{i}np)\\
I_n &= \sum_l J_l(z')J_{n-2l}(z)
\end{aligned}$$

where J_l is the lth order Bessel function. The effective magnetic potential $\widetilde{\Delta}_{\mathrm{m}}(x)$ couples the state with wave vector $\mathbf{k}$ not only to the state with wave vector $\mathbf{k}+\mathbf{Q}$, but also to an infinite set of states with quantized wave vectors $\mathbf{k}+\mathbf{Q}_n$. The nesting wave vector should be determined to get the higher critical temperature and the lower SDW phase free energy. Such a condition implies that the x-component should be quantified. The choice $\mathbf{Q}_n = \left(2k_{\mathrm{F}} + \frac{n}{x_0}, Q_y\right)$ insures that the effective potential opens a gap at the Fermi level in the quasiparticle spectrum. It can be also shown that, for these quantified wave vectors, the magnetic response function exhibits a logarithmic divergence as $T \to 0$, characteristic of one dimensional systems. The one-dimensional nature of the problem is obvious from equations (8, 9, 10). We still have to determine the y-component of the wave vector. This can be done numerically by minimising the SDW free energy for each value of the field.

$\widetilde{\Delta}_{\rm m}$ opens in the quasiparticle spectrum an infinite number of gaps separated by the cyclotron energy $\hbar\omega_{\rm c} = \frac{eHv_{\rm F}b}{c}$. Because of the wave vector quantization, one of these gaps is always opened at the Fermi level: this is the main gap. The quantum number n is determined by the condition that the main gap is the largest one. The other secondary gaps have smaller effects on the physical properties of the system provided $\Delta_{\rm m} \ll \hbar\omega_{\rm c}$ and will be neglected here. Furthermore, we shall only consider the case of an applied magnetic field much smaller than the thermodynamic critical field for superconductivity. The effects on $\Delta_{\rm m}$ can be large when $\hbar\omega_{\rm c}$ is of the order of t', while those on $\Delta_{\rm s}$ are important only when $\hbar\omega_{\rm c} \simeq t$. For that reason, we shall ignore the orbital effect of the field on $\Delta_{\rm s}$.

With these approximations, we obtain superconductivity and SDW Green's functions as a function of superconductivity and SDW order parameters

$$F_{\rm m}^{+}(k,\omega_n) = \frac{\hbar\Delta_{\rm m}^{*}(k)}{(\hbar\omega_n)^2 + {\xi^0}^2(k_x) + \Delta_{\rm s}^2(k) + \Delta_{\rm m}^2(k)} \quad (14)$$

$$F_{\rm s}^{+}(k,\omega_n) = \frac{\hbar\Delta_{\rm s}^{*}(k)}{(\hbar\omega_n)^2 + {\xi^0}^2(k_x) + \Delta_{\rm s}^2(k) + \Delta_{\rm m}^2(k)} \cdot \quad (15)$$

It should be emphasized that only the one-dimensional dispersion relation $\xi^0\,(k_x)$ enters these equations.

We now turn to the superconductivity and SDW gap equations which are given by the self-consistent conditions

$$\Delta_{\rm s}(k) = \sum_{k'} g_{\rm s}(k,k')\langle c_{H,-k'\downarrow} c_{H,k'\uparrow}\rangle \quad (16)$$

$$\Delta_{\rm m}(k) = -\sum_{k'} g_{\rm m}^{\rm eff}(k+q,k',H)\langle c_{H,k'+q\downarrow}^{+} c_{H,k'\uparrow}\rangle. \quad (17)$$

Here the magnetic coupling $g_{\rm m}^{\rm eff}$ is due to the orbital effect of the field which changes $\Delta_{\rm m}$ in $\widetilde{\Delta}_{\rm m}$. Keeping in the equation (13) only the term corresponding to the main gap, we obtain the following effective coupling constant

$$g_{\rm m}^{\rm eff}(k+q,k',H) = g_{\rm m}(k,k')I_N^2\,(H,T) \quad (18)$$

where $N\,(H,T)$ is an integer number given by

$$I_N^2\,(H,T) = \max_n\left[I_n^2\,(H,T)\right]. \quad (19)$$

Using (14, 15, 16, 17) we obtain

$$\Delta_{\rm s}(k) = \frac{1}{\beta}\sum_{k'} g_{\rm s}(k,k')\sum_n {\rm e}^{-{\rm i}\omega_n\epsilon} \times \frac{\Delta_{\rm s}(k')}{(\hbar\omega_n)^2 + {\xi^0}^2(k') + \Delta_{\rm s}^2(k') + {\Delta_{\rm m}}^2(k')} \quad (20)$$

and

$$\Delta_{\rm m}(k) = \frac{1}{\beta}\sum_{k'} g_{\rm m}^{\rm eff}(k,k',H)\sum_n {\rm e}^{-{\rm i}w_n\epsilon} \times \frac{\Delta_{\rm m}(k')}{(\hbar w_n)^2 + {\xi^0}^2(k') + \Delta_{\rm s}^2(k') + \Delta_{\rm m}^2(k')} \cdot \quad (21)$$

Angle-resolved photoemission spectroscopy (ARPES), penetration depth measurements, point contact tunneling measurements and Raman scattering experiments were all interpreted as providing evidence for a superconducting order parameter consistent with $d_{x^2-y^2}$ symmetry [1,4,11–17]. For that reason, we have choosen to impose such a symmetry for the superconducting order parameter in the solution of the gap equations. As regards the SDW order parameter, its symmetry is imposed by the numerical solutions of the coupled self-consistent equations. On the other hand, it is required that the SDW order parameter does not vanish on the node of the $d_{x^2-y^2}$–wave superconductivity order parameter [10]. The SDW order parameter considered here corresponds to the d_{xy}–wave symmetry.

Such a choice for the symmetry of the SDW order parameter is not obvious. We have also tried different simple k-dependences [18,19], in particular a k-independent constant for the magnetic gap. It turned out that the d_{xy} –wave symmetry gives the higher stability, among the other simple choices we have made. This is probably due to the fact that the magnetic gap is maximum at the nodes of the superconducting order parameter. Of course, we cannot exclude that more complex solutions would give a lower free energy.

The next step consists in defining superconductivity and SDW coupling potentials which display the gap symmetry. The approximated expressions are given by

$$g_{\rm s}(k,k') = g_{\rm os} f_{\rm s}(k) f_{\rm s}(k')$$
$$g_{\rm m}^{\rm eff}(k,k',H) = g_{\rm om}^{\rm eff}(H) f_{\rm m}(k) f_{\rm m}(k')$$

where

$$f_{\rm s}(k) = \cos(k_x a) - \cos(k_y b)$$
$$f_{\rm m}(k) = \sin(k_x a)\sin(k_y b).$$

The field dependence of $g_{\rm om}^{\rm eff}$ is due to the orbital effect of the field, responsible for quantum interference effects analyzed in details in reference [9]. This effect induces better nesting properties and, therefore, an effective coupling constant $g_{\rm om}^{\rm eff}(H)$ rapidly increasing with the field.

We next introduce the coupled gap equations which can be written as follow

$$1 = g_{\rm os}\sum_k f_{\rm s}^2(k)\frac{\tanh(\frac{E_k}{2kT})}{2E_k} \quad (22)$$

$$1 = g_{\rm om}^{\rm eff}\sum_k f_{\rm m}^2(k)\frac{\tanh(\frac{E_k}{2kT})}{2E_k} \quad (23)$$

where

$$E_k = \sqrt{{\xi^0}^2 + \Delta_{\rm s}^2(k) + \Delta_{\rm m}^2(k)}$$
$$\Delta_{\rm s}(k) = \Delta_{\rm os} f_{\rm s}(k)$$
$$\Delta_{\rm m}(k) = \Delta_{\rm om} f_{\rm m}(k).$$

2.2 Thermodynamic potential

The thermodynamic potential Ω is calculated by means of integration over a variable coupling constant [20]. We consider that the thermodynamic potential Ω_n in the normal state is just that of a free Fermi gas Ω_0. Then, the thermodynamic potential, compared with that of the metallic phase Ω_0 will be given by

$$\Omega_i - \Omega_0 = \int_0^1 \frac{\mathrm{d}\lambda}{\lambda} \langle \lambda H_i \rangle \tag{24}$$

where i indicates the corresponding state. According to (24), we have calculated the thermodynamic potential for the three possible phases: the superconducting phase, the pure SDW phase and a mixed phase in which a superconducting order parameter and a magnetic order parameter coexist. The mean field expressions are the following

$$\Omega_\mathrm{s} - \Omega_0 = \frac{\Delta_\mathrm{s}^2(k)}{g_\mathrm{os}} - \frac{2}{\beta}\sum_k \mathrm{Ln}\left[\frac{\cosh(\frac{\beta E_k^\mathrm{s}}{2})}{\cosh(\frac{\beta \xi^0(k)}{2})}\right] \tag{25}$$

$$\Omega_\mathrm{m} - \Omega_0 = \frac{\Delta_\mathrm{m}^2(k)}{g_\mathrm{om}^\mathrm{eff}} - \frac{2}{\beta}\sum_k \mathrm{Ln}\left[\frac{\cosh(\frac{\beta E_k^\mathrm{m}}{2})}{\cosh(\frac{\beta \xi^0(k)}{2})}\right] \tag{26}$$

$$\begin{aligned}\Omega_\mathrm{mixed} - \Omega_0 = & \frac{\Delta_\mathrm{s}^2(k)}{g_\mathrm{os}} + \frac{\Delta_\mathrm{m}^2(k)}{g_\mathrm{om}^\mathrm{eff}} \\ & - \frac{2}{\beta}\sum_k \mathrm{Ln}\left[\frac{\cosh(\frac{\beta E_k}{2})}{\cosh(\frac{\beta E_k^\mathrm{s}}{2})}\right] \\ & - \frac{2}{\beta}\sum_k \mathrm{Ln}\left[\frac{\cosh(\frac{\beta E_k}{2})}{\cosh(\frac{\beta E_k^\mathrm{m}}{2})}\right]\end{aligned} \tag{27}$$

where

$$E_k^\mathrm{s} = \sqrt{\xi^0(k)^2 + \Delta_\mathrm{s}^2(k)}$$
$$E_k^\mathrm{m} = \sqrt{\xi^0(k)^2 + \Delta_\mathrm{m}^2(k)}$$
$$E_k = \sqrt{\xi^0(k)^2 + \Delta_\mathrm{s}^2(k) + \Delta_\mathrm{m}^2(k)}.$$

3 Results

In order to find the most stable state, we have calculated the thermodynamic potential as a function of temperature and of the SDW coupling constant $g_\mathrm{om}^\mathrm{eff}$ (or, equivalently, of the magnetic field) for the possible phases considered in this work: a pure d-wave superconducting phase, a pure SDW phase alone and a mixed phase in which superconducting and SDW orders coexist. In that calculation, we have choosen typical plausible physical parameters describing the system: $g_\mathrm{os} = 0.5$ eV, the energy cut off is 0.01 eV for the superconducting interaction and 0.6 eV for the magnetic interaction. For such parameters, the superconducting critical temperature is fixed at 110 K.

In Figure 1 we report the thermodynamic potential dependence on the SDW coupling constant $g_\mathrm{om}^\mathrm{eff}$ of each phase, at a fixed temperature T. In Figure 1, $T = 10$ K, but the corresponding behaviour has a general validity, at least up to $T < 100$ K. This graph reveals different domains of stability for the various phases.

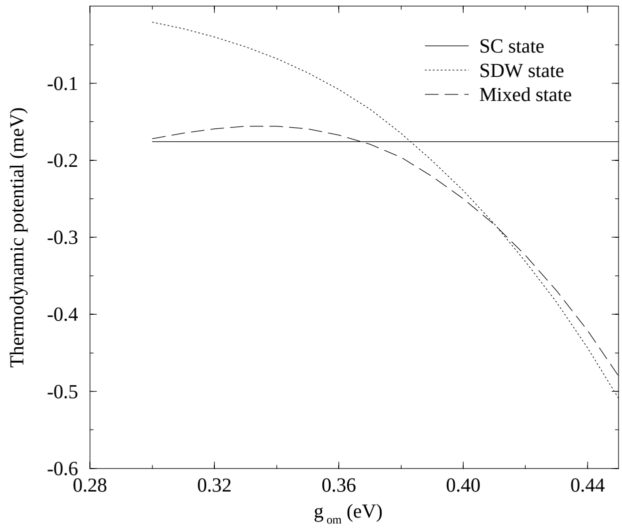

Fig. 1. Thermodynamic potential dependence on the SDW coupling constant $g_\mathrm{om}^\mathrm{eff}$ of the three phases: superconducting phase (Solid line), SDW phase (Dashed line) and mixed phase (Long dashed line) calculated for $g_\mathrm{os} = 0.5$ eV and $T = 10$ K.

At low values of the magnetic interaction $g_\mathrm{om}^\mathrm{eff}$, the stable phase is the pure superconducting phase. Then for increasing $g_\mathrm{om}^\mathrm{eff}$, the mixed phase becomes stable for $g_\mathrm{om}^\mathrm{eff}$ varying in the range $g_\mathrm{om1}^\mathrm{eff} < g_\mathrm{om}^\mathrm{eff} < g_\mathrm{om2}^\mathrm{eff}$. At $g_\mathrm{om}^\mathrm{eff} = g_\mathrm{om1}^\mathrm{eff}$ a phase transition from the superconducting phase to the mixed phase occurs. Then, at $g_\mathrm{om}^\mathrm{eff} = g_\mathrm{om2}^\mathrm{eff}$ a second transition occurs, from the mixed phase to the field-induced SDW phase.

In Figures 2, 3 and 4 we show the temperature dependence of the thermodynamic potentials for the three phases as a function of temperature for three typical values of $g_\mathrm{om}^\mathrm{eff}$. In Figure 2, $g_\mathrm{om}^\mathrm{eff} = 0.36$ eV: the superconducting phase is stable at low temperature, up to a critical temperature ($T = 22$ K) at which a transition to the mixed phase occurs; The second transition, from the mixed state to the superconducting state, occurs at the temperature $T = 80$ K. In Figure 3, for a larger value of $g_\mathrm{om}^\mathrm{eff} = 0.38$ eV, the mixed phase is the most stable. In Figure 4, for $g_\mathrm{om}^\mathrm{eff} = 0.40$ eV, the mixed phase is stable until $T = 100$ K, where it degenerates with SDW phase.

The study of the T and $g_\mathrm{om}^\mathrm{eff}$ dependence of the thermodynamic potentials allows to propose a phase diagram shown in Figure 5, valid for the set of parameters choosen above. The curves $g_\mathrm{om1}^\mathrm{eff}(T)$ and $g_\mathrm{om2}^\mathrm{eff}(T)$ indicate the first order transition lines separating the domains of stability of the three phases.

Then, we show the behaviour of the different order parameters, as a function of temperature and field. In Figures 6 and 7 we report numerical solutions of equations (22, 23), in the mixed phase, for different values of $g_\mathrm{om}^\mathrm{eff}$ and for $T = 10$ K.

As obvious from these figures, the SDW order parameter Δ_om increases with $g_\mathrm{om}^\mathrm{eff}$ (and therefore with

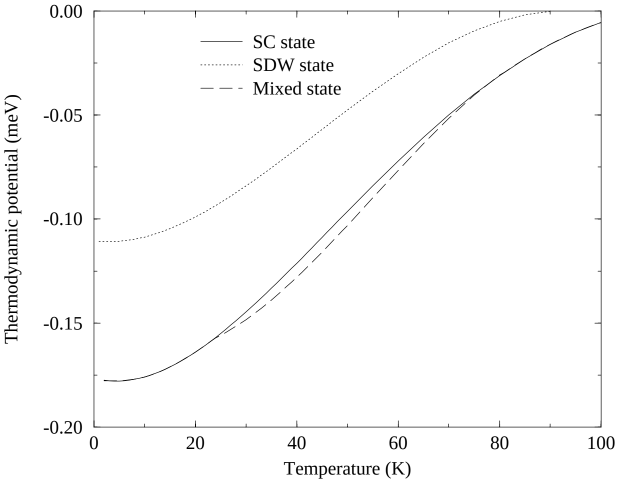

Fig. 2. Thermodynamic potential dependence on temperature of the three phases: superconducting phase (solid line), SDW phase (dashed line) and mixed phase (long dashed line) calculated for $g_{\rm os} = 0.5$ eV and $g^{\rm eff}_{\rm om} = 0.36$ eV.

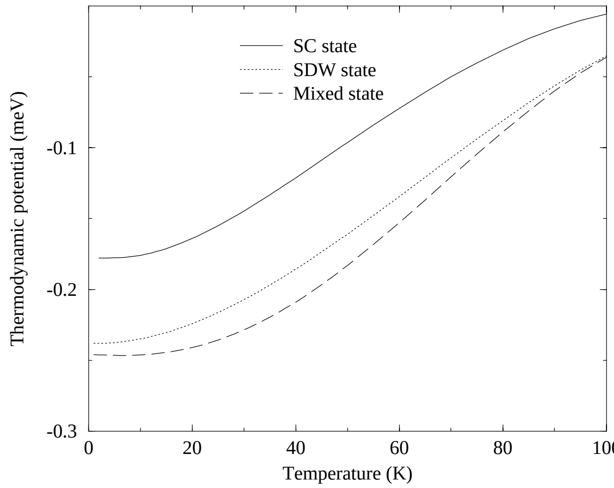

Fig. 4. Thermodynamic potential dependence on temperature of the three phases: superconducting phase (solid line), SDW phase (dashed line) and mixed phase (long dashed line) calculated for $g_{\rm os} = 0.5$ eV and $g^{\rm eff}_{\rm om} = 0.40$ eV.

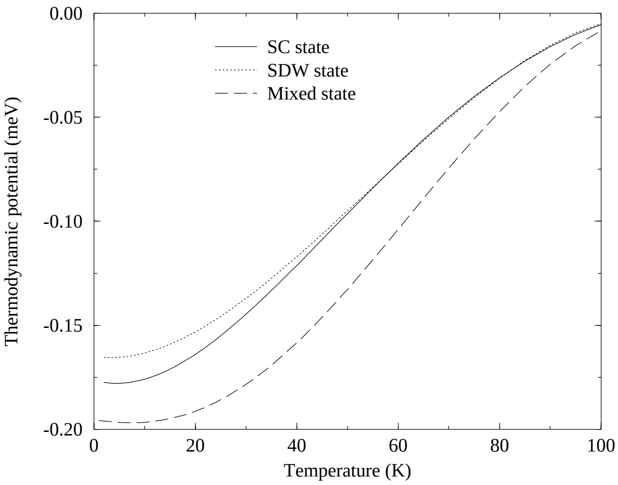

Fig. 3. Thermodynamic potential dependence on temperature of the three phases: superconducting phase (solid line), SDW phase (dashed line) and mixed phase (long dashed line) calculated for $g_{\rm os} = 0.5$ eV and $g^{\rm eff}_{\rm om} = 0.38$ eV.

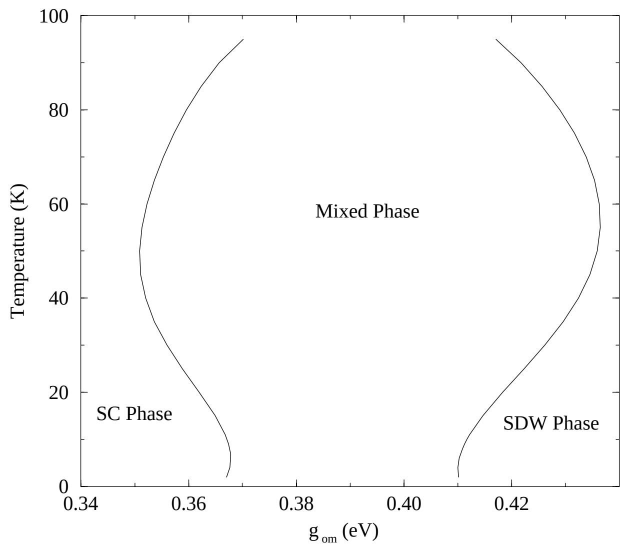

Fig. 5. Phase diagram corresponding to the dependence of $g^{\rm eff}_{\rm om1}$ and $g^{\rm eff}_{\rm om2}$ on temperature for $g_{\rm os} = 0.5$ eV.

the field). In the same time the superconductivity order parameter $\Delta_{\rm os}$ is a decreasing function of $g^{\rm eff}_{\rm om}$. We remark that the transition from $d_{x^2-y^2}$ superconductivity to the mixed phase, where superconductivity and SDW coexist, is a first order, as already shown by the finite angles at which the thermodynamic potential curves cross in Figures 2, 3 and 4. The transition line separating the mixed phase and the SDW phase is also characterized by a first order transition.

In Figures 8 and 9, we illustrate the superconducting and SDW order parameters dependence on temperature, in the mixed phase, for different values of the SDW coupling constant $g^{\rm eff}_{\rm om}$. The two order parameters are calculated in the mixed phase for $g^{\rm eff}_{\rm om} = 0.38$ eV and $g^{\rm eff}_{\rm om} = 0.40$ eV. The corresponding variation of the thermodynamic potentials of the three phases were shown in Figures 3 and 4. From these figures, we can see that, in the mixed phase, with increasing values of $g^{\rm eff}_{\rm om}$, the superconductivity order parameter decreases and the SDW one increases.

The most important conclusion of this study, in particular from the phase diagram of Figure 5, is that a magnetic field applied perpendicularly to the conducting plane can induce a first order phase transition to a mixed phase in which superconductivity and magnetism coexist. Such a phase corresponds to a new symmetry breaking, since the time reversal symmetry as well as the gauge symmetry are broken. Consequently, a gap is open over the whole Fermi surface. The quasiparticles existing at low temperature in the pure d-wave superconducting phase, because of the nodes of the gap, should, therefore, disappear discontinuously at the critical field. This might be the explanation of the plateau observed in the thermal conductivity κ of Bi2212 above a threshold applied field [5]. In

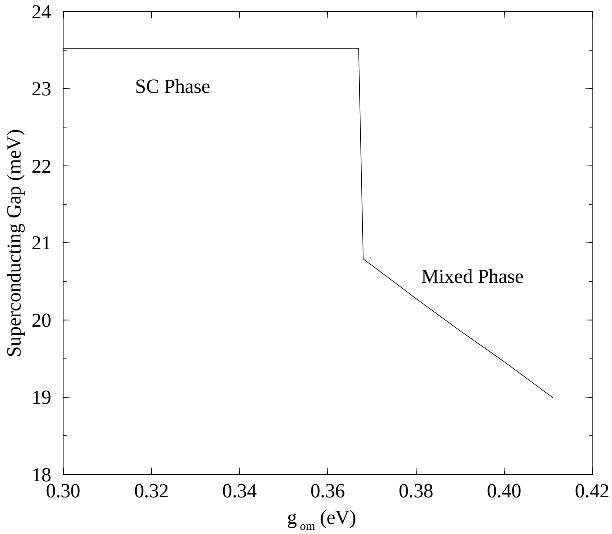

Fig. 6. Superconductivity order parameter as a function of $g_{\rm om}^{\rm eff}$ for $g_{\rm os} = 0.5$ eV and $T = 10$ K.

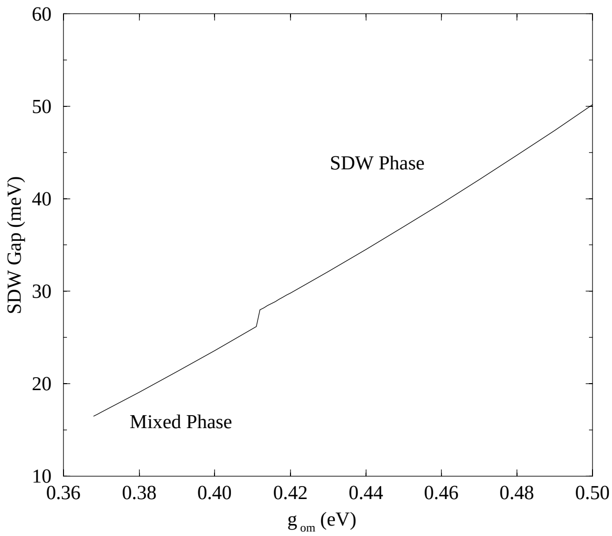

Fig. 7. SDW order parameter as a function of $g_{\rm om}^{\rm eff}$ for $g_{\rm os} =$ 0.5 eV and $T = 10$ K.

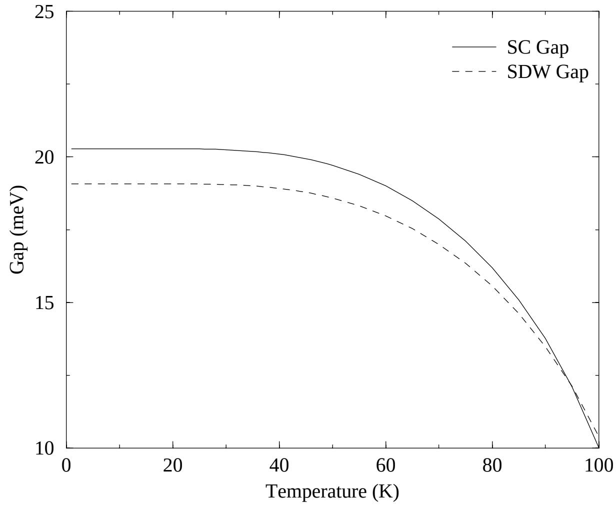

Fig. 8. Superconductivity (solid line) and SDW (dashed line) order parameters as a function of temperature calculated for $g_{\rm os} = 0.5$ eV and $g_{\rm om}^{\rm eff} = 0.38$ eV.

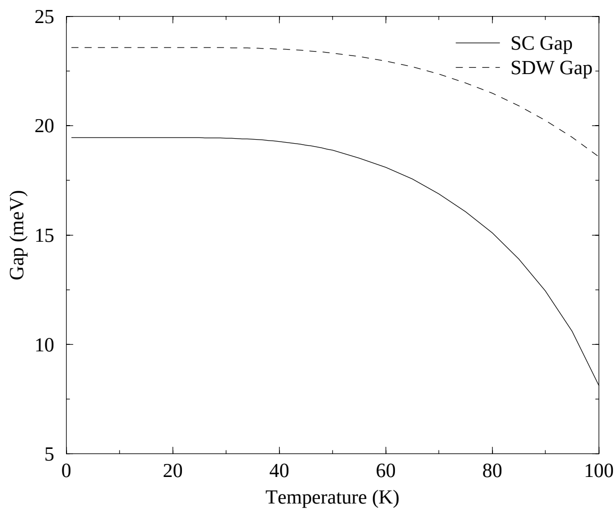

Fig. 9. Superconductivity (solid line) and SDW (dashed line) order parameters as a function of temperature calculated for $g_{\rm os} = 0.5$ eV and $g_{\rm om}^{\rm eff} = 0.40$ eV.

this interpretation, the sharp break in slope in the thermal conductivity is a signature of the first order transition and the flat plateau that extends to high field is the signature of the mixed phase, in which the quasiparticle current vanishes. Therefore, our results provide a plausible interpretation of the experiments [5].

4 Discussion

In this section we have to start with a brief discuss of the recent work including high temperature superconductors.

It could be thought, at first sight, that the kink in the thermal conductivity as a function of the applied field implies a second order phase transition. Such is not the case, as already discussed by various authors, such as Krishana*et al.* [5], Lee [21] and Laughlin [10]. If the observed plateau in the thermal conductivity is an experimental evidence for the disappearance of quasiparticles, this means that the quasiparticle density vanishes abruptly at the critical field, in spite of the finite temperature. This can be the case only if the field induces a large gap in the excitation spectrum, of the order of the temperature or larger. Therefore, the anomaly in the thermal conductivity should be the signature of a strongly first order transition.

In a recent paper, Laughlin [10], attributed the transition observed in Bi2212 by Krishana *et al.* [5] to the development of a small d_{xy} superconducting order parameter phased by $\frac{\pi}{2}$ with respect to the principal $d_{x^2-y^2}$ one to produce a minimum energy gap. Laughlin argued that the new high-field state is the parity and time-reversal violating $d_{x^2-y^2} + {\rm i}d_{xy}$ superconducting state [22]. The essential point of this argument is that the state must have a magnetic moment in order to account for the experiment and this is possible only if it violates both parity and time-reversal symmetry.

Recently, Anderson [23] attributed the anomalously sharp dependence of the thermal conductivity on magnetic field in Bi2212 to the appearance of magnetic field-induced discrete spectrum at the gap nodes. First, Anderson suggests that the magnetic field cause a Larmor precession of all the electrons around the Fermi surface and shows that Larmor frequency remains small compared with the maximum gap. In order to go beyond this latter semiclassical picture, Anderson solve the Bogoliubov-de Gennes equations for the quasi-particle wave function near one of the k-space nodes of the gap. Space dependence of the gap, where the vortices are dense enough to overlap strongly so that the magnetic field is uniform, was included.

We have implicitly assumed, up to now, a uniform penetration of the magnetic field in the bulk of the sample. This is far from obvious, since the magnetic flux is expelled by the Meissner effect. In fact, the field penetrates in the sample through the vortex lines. Since Bi2212 is an extreme type II superconductor, the London penetration length λ is quite large (much larger than the coherence length), so that, for the field used by Krishana *et al.* [5], the mean distance between vortices is smaller than λ. Therefore, for these fields, the magnetic field can be considered as uniform over the sample. Let us note that, whatever the correct interpretation of the phenomena may be, the field penetration in the sample is due to the vortices. Therefore, as the field intensity is varied, one should expect hysteresis and irreversibility associated to the vortex dynamics. This has been observed and commented recently by Behnia *et al.* [24]. If, for large fields, we can consider the penetration as nearly uniform, obviously, this is no longer the case at low field, when the mean distance between vortices becomes larger than λ. In such a case, one would not expect a long range order in the mixed phase, but rather regions of short range order around the vortices, with a magnetic correlation length of the order of λ. The discontinuities associated with the phase transitions should be smoothed and eventually disappear. Let use note that Behnia *et al.* [25] have reported the absence of anomalies in the low-temperature-low-field part of the phase diagram. This can be due to the effect discussed above, which is by no means in contradiction with Behnia's comment, which argued that these data imply the presence of quasi-particles in this part of the phase diagram.

Of course, we cannot discard the possibility that the anomalies observed in the thermal conductivity κ have a completely different origin. In particular, it is possible that these discontinuities might be related, not to an electronic phase transition, but to another completely different phenomenon, such as the hysteretic behaviour of the magnetization in the superconducting phase, because of the irreversible vortex dynamics [26].

As discussed above, the $g_{\rm m}$ axis can be considered as a magnetic field axis, since the strength of the effective interaction can be varied by varying the strength of the field, through equations (18, 19). The coefficient I_N, in our simple model, is a unique function of $\frac{t'}{H}$. Therefore, for a given value of t', we are able to determine the field range of existence of the mixed phase. Unfortunately, no precise evaluation of the exact value of t' in superconductors as Bi2212 is still available, neither by *ab initio* calculation, nor by experimental determination. Such an evaluation of t' is made difficult by its small value, much smaller than t. If we consider as plausible value of $t' \simeq 10$ meV, then the mixed phase should be stable for field larger than $H_1 \simeq 40$ T, but smaller than $H_2 \simeq 60$ T. For $t' \simeq 1$ meV, then $H_1 = 4$ T and $H_2 = 6$ T.

In this work, we have neglected the diamagnetic effect on the thermodynamic potential of the superconducting phase. This approximation is justified for fields much smaller than the upper critical field H_{c2}. This condition should be fulfilled in the relevant part of the phase diagram in the (H, T) plane for small enough values of t'. However, for larger values, this is no longer the case and the diamagnetic term should be included in the superconducting phase thermodynamic potential. We have not taken into account this effect which should favour the mixed phase. We have not either taken into account in details the role of the vortices in the Abrikosov phase of the d-wave superconductor. The Doppler shift effects due to circulating supercurrents around the vortices produce, in the Abrikosov phase, a finite density of states at the Fermi energy, corresponding to those excitations that have move down in energy. But this effect disappears when the magnetic order sets in, because of the presence of a large SDW gap. Therefore, this effect, which has been neglected here, is expected to enhance the stability of the coexistence phase, since it weakens the superconducting gap. The detailed study of this effect will be devoted to a forthcoming paper.

5 Conclusion

In summary, we have performed a mean field treatment of a simple model of quasi-two-dimensional d-wave superconductor, assuming two possible order parameters: a superconducting one and a magnetic one. We have shown that a magnetic field can induce a mixed phase in which both orders coexist. Such a new phase corresponds to a new symmetry breaking and should exhibit a finite gap over the whole Fermi surface, at variance with the zero field phase, in which the gap exhibits nodes on the Fermi surface. We propose that such a simple model might provide a possible explanation for the anomalies observed in the thermal conductivity of Bi2212 in a moderate applied field.

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